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WAVE MOTION OF SOLID PARTICLES IN A PULSED GAS FLOW

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The effect of low-frequency pulsations of gas on the motion of solid particles has been studied on the basis of numerical solution of equations of the dynamics of a monodisperse gas suspension with account for interphase forces of hydrodynamic drag, virtual masses, and forces due to nonstationary effects around particles. It is found that at certain parameters gas pulsations lead to enhancement of interphase heat transfer. The dependences of the time of particle residence in a pneumochannel on the frequency of gas pulsations have been obtained.

A thermodynamic mode of phase interaction exerts a crucial effect on the intensity of heat- and mass-transfer processes and the efficiency of equipment operation. Fluctuations of the carrying phase can substantially affect the hydrodynamics and interphase heat and mass transfer in disperse systems. At certain parameters, nonstationary flows lead to enhancement of a number of technological processes, e.g., dissolution, extraction, crystallization, combustion, etc. Heat- and mass-transfer enhancement arises in an outer high-velocity pulsed gas flow past particles. In calculation and designing of equipment it is important to know the laws governing the effect of fluctuations of the carrying phase on the intensity of heat- and mass-transfer processes.

Recently, discrete-pulsed nonstationary modes of energy input to disperse systems and nonstationary wave and resonance modes of the carrying-phase flow with a large amplitude of velocity and pressure fluctuations have been developed. Different devices are used to produce medium pulsations. One effective generator of high-temperature pulsed gas flows is the intermittent combustion chamber. These nonstationary flows can be used for implementation of energy-conservation technologies of drying disperse materials and solutions [1–4].

A suspended state of the dispersed phase is provided by hydrodynamic drag forces. In the case of a constant velocity of motion of particles, they are affected by forces caused by the pressure gradient, the difference of velocities, and phase densities. In wave motion of particles, the forces caused by the nonstationary character of phase motion are added. Among these are the force of virtual masses due to inertia effects and the "hereditary" Basset force, which arises as a result of nonstationary effects in the carrying phase (nonstationarity of the boundary layer around particles). A great many works [5–14] presenting the results of investigations of pulsed devices and the effect of harmonic fluctuations of the carrying phase (gas or liquid) on motion and heat and mass transfer of particles are known. It should be noted that, for the most part, these results are valid only for the case of motion of individual particles. The condition of smallness of the volumetric concentration of particles ($n_2 \to 1$) is more strict compared to the case of the absence of particle interaction; for a monodisperse mixture it can be restricted by a value of $\varepsilon_2 \le 0.3$ [6]. It is noteworthy that the results of the investigations of a pulsed motion of particles in liquids cannot be generalized directly to the motion and heat and mass transfer in a pulsed gas flow with a large amplitude of velocity fluctuations.

We consider the wave motion of a monodisperse mixture in the direction opposite to the action of the gravity force provided that the velocity of the carrying phase (gas) changes according to the harmonic dependence

$$v_1 = \bar{v}_1 + v_1^A \sin(\omega t) . \tag{1}$$

We consider the case of the absence of phase conversions; interaction of particles can be neglected due to the fact that the volumetric concentration of particles is not very large. Within the framework of the model of interpenetrating continua, the volumetric concentration of the solid phase is presented as a "frozen" volumetric concentration

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ε_2. The quantity ε_1 corresponds to porosity. We assume that the barotropy condition holds, i.e., $P = P_1 = P_2$; this condition takes place in most cases in practice.

With account for the assumptions made, the equations of nonstationary wave motion are written as follows:

$$\rho_1\varepsilon_1 \frac{dv_1}{dt} = -\rho_1\varepsilon_1 g - \varepsilon_1 \frac{\partial P}{\partial x} - F_\mu - F_\mathrm{m} - F_\mathrm{B} , \tag{2}$$

$$\rho_2\varepsilon_2 \frac{dv_2}{dt} = -\rho_2\varepsilon_2 g - \varepsilon_2 \frac{\partial P}{\partial x} + F_\mu + F_\mathrm{m} + F_\mathrm{B} , \tag{3}$$

$$\varepsilon_1 + \varepsilon_2 = 1 . \tag{4}$$

The hydrodynamic drag force for an individual spherical particle is

$$F_\mu^0 = \xi S_2 \frac{\rho_1 \left| v_1 - v_2 \right| (v_1 - v_2)}{2} = \xi \frac{\pi d^2}{8} \rho_1 \left| v_1 - v_2 \right| (v_1 - v_2) . \tag{5}$$

The number of particles per volume unit is

$$n_2 = \frac{6\varepsilon_2}{\pi d^3} . \tag{6}$$

Then, the hydrodynamic drag force for the disperse mixture is written as

$$F_\mu = \frac{3}{4} \frac{\varepsilon_2}{d} \xi \rho_1 \left| v_1 - v_2 \right| (v_1 - v_2) . \tag{7}$$

The force of virtual masses for a particle is determined by the expression

$$F_\mathrm{m}^0 = \frac{1}{12} \pi d^3 \rho_1 \frac{\partial}{\partial t} (v_1 - v_2) , \tag{8}$$

for the disperse mixture it is

$$F_\mathrm{m} = \frac{n_2}{12} \pi d^3 \rho_1 \frac{\partial}{\partial t} (v_1 - v_2) = \frac{1}{2} \varepsilon_2 \rho_1 \frac{\partial}{\partial t} (v_1 - v_2) . \tag{9}$$

The force caused by the nonstationarity of a viscous boundary layer around particles (the "hereditary" Basset force) is found from [14]:

$$F_\mathrm{B}^0 = \frac{3d^2}{2} \sqrt{\pi \rho_1 \mu_1} \int_0^t \frac{\partial}{\partial \tau} (v_1 - v_2) \frac{d\tau}{\sqrt{t - \tau}} ; \tag{10}$$

for the disperse mixture it is

$$F_\mathrm{B} = \frac{9\varepsilon_2}{\pi d} \sqrt{\pi \rho_1 \mu_1} \int_0^t \frac{\partial}{\partial \tau} (v_1 - v_2) \frac{d\tau}{\sqrt{t - \tau}} . \tag{11}$$

Calculation of the integral entering in the expression for the Basset force complicates the solution of the problem. Following [13], to calculate the integral we use the theorem of the mean. Allowing for the periodic character of motion, we take the length of one period as the upper limit of integration in expression (11). Then,

$$\int_0^{t'} \frac{d\,(v_1 - v_2)}{\sqrt{t' - \tau}} = \frac{v_1 - v_1\,(0) - v_2 + v_2\,(0)}{\sqrt{t' - \tau_*}} \,. \tag{12}$$

Here $0 < \tau_* < t'$. The mean time in first approximation is taken to be equal to $\tau_* = t'/2$. Since $t' = 1/f$ and $\omega = 2\pi f$, we obtain

$$F_{\rm B} = \frac{9\varepsilon_2 \sqrt{\rho_1 \mu_1 \omega}}{\pi d} \left[v_1 - v_1\,(0) - v_2 + v_2\,(0)\right] . \tag{13}$$

Having expressed the pressure gradient from Eq. (2)

$$\frac{\partial P}{\partial x} = -\,\rho_1 \frac{dv_1}{dt} - \rho_1 g - \frac{1}{\varepsilon_1} (F_\mu + F_{\rm m} + F_{\rm B}) \tag{14}$$

and substituted it in (3), we have

$$\rho_2 \varepsilon_2 \frac{dv_2}{dt} = -\,(\rho_2 - \rho_1)\,\varepsilon_2 g + \rho_1 \varepsilon_2 \frac{dv_1}{dt} + \frac{1}{\varepsilon_1} (F_\mu + F_{\rm m} + F_{\rm B}) \tag{15}$$

or

$$\rho_2 \varepsilon_1 \frac{dv_2}{dt} = -\,(\rho_2 - \rho_1)\,\varepsilon_1 g + \rho_1 \varepsilon_1 \frac{dv_1}{dt} + \frac{1}{\varepsilon_2} (F_\mu + F_{\rm m} + F_{\rm B}) \,. \tag{16}$$

Then, substituting expressions (7), (9), and (13) in (16), we obtain

$$\frac{dv_2}{dt} = -\frac{2\,(\rho_2 - \rho_1)\,\varepsilon_1}{2\rho_2 \varepsilon_1 + \rho_1} g + \frac{2\varepsilon_1 \rho_1 + \rho_1}{2\rho_2 \varepsilon_1 + \rho_1} \frac{dv_1}{dt} + \frac{3}{2} \frac{\xi}{d} \frac{\rho_1}{2\rho_2 \varepsilon_1 + \rho_1} \left| v_1 - v_2 \right| (v_1 - v_2) +$$

$$+ \frac{18}{\pi d} \frac{\sqrt{2\pi \rho_1 \mu_1 f}}{2\rho_2 \varepsilon_1 + \rho_1} \left[v_1 - v_1\,(0) - v_2 + v_2\,(0)\right] , \tag{17}$$

$$\frac{dx}{dt} = v_2 \,. \tag{18}$$

Here the hydromechanic drag is determined with account for motion constraint by the dependences suggested by M. A. Gol'dshtik:

$$\xi = C^2 \xi^0\,({\rm Re}^*) \,, \tag{19}$$

$$\xi^0\,({\rm Re}^*) = \frac{24}{{\rm Re}^*} + 0.248 \left(1 + \sqrt{1 + \frac{194}{{\rm Re}^*}}\right), \tag{20}$$

$${\rm Re}^* = C\,{\rm Re} \,, \tag{21}$$

$$C = \frac{\varepsilon_1}{1 - 1.16\varepsilon_2^{2/3}} \,. \tag{22}$$

We consider low-frequency gas pulsations when the amplitude of medium displacement A is much larger than the diameter of solid particles and the flow past the latter can be taken as quasi-stationary, i.e., the field of gas velocities at each time instant obeys the laws governing a stationary flow. The displacement amplitude is related to the amplitude of the vibrational speed of gas and frequency as $A = v_1^A/\omega$. Let $v_1^A = 10$ m/sec and $f = 100$ Hz; consequently, the displacement amplitude is 0.016 m, and for solid particles with a diameter of $1 \cdot 10^{-4}$ m, $A/d >> 1$.

The equation of energy conservation is

$$\frac{d(\varepsilon_2 \rho_2 e_2)}{dt} = Q_2 , \tag{23}$$

where $e_2 = c_2 T_2$.

The heat flux to the particle is

$$q_2 = \pi d \lambda_1 \,\mathrm{Nu}\,(T_1 - T_2) ; \tag{24}$$

then

$$Q_2 = n_2 q_2 = \frac{6\varepsilon_2}{d^2} \lambda_1 \,\mathrm{Nu}\,(T_1 - T_2) . \tag{25}$$

The equation of heat-transfer kinetics is

$$\frac{dT_2}{dt} = \frac{6\lambda_1 \,\mathrm{Nu}}{\rho_2 c_2 d^2} (T_1 - T_2) . \tag{26}$$

As has already been mentioned, in this case, the process of heat transfer can be considered as quasi-stationary and the Nusselt number can be found from the dependence $\mathrm{Nu} = 2 + 0.55\,\mathrm{Re}^{0.5}\,\mathrm{Pr}^{0.33}$.

The initial conditions are

$$t = 0 , \;\; x = 0 , \;\; v_2 = \frac{dx}{dt} = 0 , \;\; T_2 = T_{20} , \;\; T_1 = T_{10} , \;\; v_1 = v_1(0) . \tag{27}$$

The time-averaged Nusselt number was determined by the expression

$$\overline{\mathrm{Nu}} = \frac{1}{\Delta t} \int_{\Delta t} \mathrm{Nu}\, dt . \tag{28}$$

The differential equations were solved by the Runge–Kutta method with a time step chosen automatically. The main initial parameters are $\rho_1 = 0.746$ kg/m^3, $\rho_2 = 1800$ kg/m^3, $T_1 = 473$ K, $T_{20} = 293$ K, $\varepsilon_1 = 0.99$, $c_2 = 1200$ J/(kg·K), and $\lambda_2 = 0.3$ W/(m·K). The calculations are made for different parameters: particle diameter $(0.02–1) \cdot 10^{-3}$ m, frequency of gas pulsations 0.0001–500 Hz, amplitude of gas-velocity fluctuations 0–60 m/sec, and mean gas velocity 2–60 m/sec.

We consider nonstationary motion of solid particles in a pulsed gas flow, when its velocity is a periodic function of time (1), and study the effect of gas pulsations on the motion and heat transfer of the particles.

We estimate the contribution of forces affecting solid particles in a pulsed gas flow. The ratio of the forces F_μ, F_m, and F_B depends on the frequency and is determined by the ratio of the characteristic time of settling of the quasi-stationary velocity field of the gas around a particle ($t_\mu = d^2/\nu_1$) to the characteristic time of variation of the gas parameters ($t_\omega = 1/\omega$).

Based on [6], we estimate the components of the interphase force. Let the following parameters be set: $T_1 = 473$ K, $\rho_1 = 0.75$ kg/m^3, $\mu_1 = 26 \cdot 10^{-5}$ Pa·sec, $\omega = 2\pi f = 2 \cdot 3.14 \cdot 100 = 628$ 1/sec. Then, $t_\mu = 0.75 d^2/26 \cdot 10^{-5} = 28{,}846 d^2$. At $d = 1$ μm $\sqrt{\omega t_\mu} = 0.0043$, at $d = 10$ μm $\sqrt{\omega t_\mu} = 0.043$, at $d = 100$ μm $\sqrt{\omega t_\mu} = 0.42$, and at $d = 1$ mm $\sqrt{\omega t_\mu} = 4.2$. It is seen from this analysis that as the particle size increases (the frequency of gas pulsations is constant), the effect of virtual masses and the "hereditary" Basset force increases.

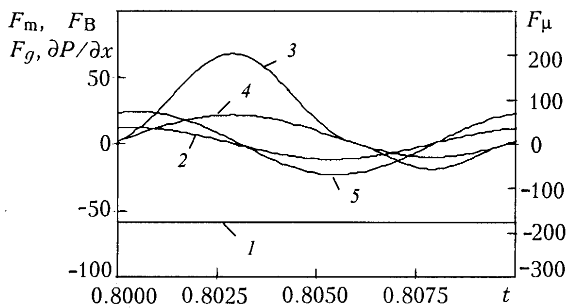

Fig. 1. Dependence of forces affecting the particles on time during the period of gas oscillations ($\bar{v}_1$ = 2 m/sec, v_1^A = 5 m/sec, ρ_2 = 600 kg/m^3, $d = 1{\cdot}10^{-3}$ m, f = 100 Hz): 1) F_g; 2) F_m; 3) F_μ; 4) F_B; 5) $\partial P/\partial x$.

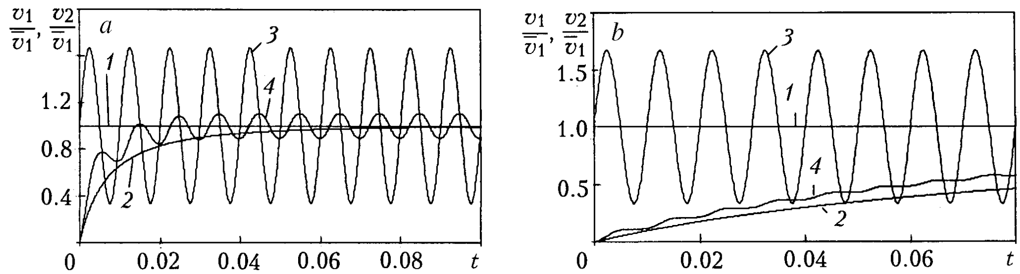

Fig. 2. Dependence of the velocity of gas (1, 3) and particles (2, 4) on time ($\bar{v}_1$ = 60 m/sec): 1 and 2) v_1^A = 0, 3 and 4) 40 m/sec; a) $d = 1{\cdot}10^{-4}$; b) $1{\cdot}10^{-3}$ m.

It should be noted that the estimate of the contribution of forces affecting the particles given in [6] is made on the basis of the consideration of propagation of small monochromatic waves in disperse suspensions. As the relative Reynolds number Re = $d|v_1 - v_2|/\nu_1$, i.e., relative velocity of flow past particles, increases, the prevailing effect on the particle motion is exerted by inertia effects and the influence of nonstationary ("hereditary") effects in the carrying phase is weak.

We calculated the values of the components F_μ, F_m, and F_B, $\partial P/\partial x$ of the interphase force, and $F_g = -\rho_2\varepsilon_2 g$ at different particle diameters in the steady-state wave motion of the gas suspension. It is found that at $\bar{v}_1$ = 60 m/sec, v_1^A = 40 m/sec, ρ_2 = 1800 kg/m^3, and f = 100 Hz with an increase in the particle diameter from 100 μm to 1 mm, the ratio of amplitudes F_m/F_μ increases from 0.11 to 1.7%, and the ratio F_B/F_μ from ~1.5 to 2.5%. The hydrodynamic drag force F_μ is much larger than the Basset force, the force of virtual masses, the gravity force, and the force caused by the pressure gradient. This is due to the large value of the relative velocity of motion of phases in the considered case, Re = $d|v_1 - v_2|/\nu_1 \approx 10^{-4}{\cdot}|100 - 60|/34.7{\cdot}10^{-6}$ = 115. However, at $\bar{v}_1$ = 2 m/sec, v_1^A = 5 m/sec, ρ_2 = 600 kg/m^3, f = 100 Hz, and d = 1 mm, the ratio of the amplitudes F_m/F_μ reaches 5.7%, and F_B/F_μ is 10.6% and 17.5% at positive and negative values, respectively. Figure 1 presents the dependences of the components of the interphase force on time during the period of gas-phase fluctuations. It is seen that the Basset force affecting the particles is in phase with the hydrodynamic drag force and the force of virtual masses is $\pi/2$ in anti-phase.

In the pulsed gas flow, the dispersed particles execute wavy motion; in this case, the modulus of the relative velocity of phases $|v_1 - v_2|$ takes maximum values in the wave loop (Fig. 2). The relative velocity of the phase motion increases compared with the motion of the particles in a gas stationary flow, since, in the latter case, the difference of phase velocities is substantial only in the accelerating section. As a result, large-amplitude oscillations of gas lead to about a twofold increase of the mean Nusselt number ($\overline{\text{Nu}}$) and the heat-transfer coefficient. This is clearly demonstrated in Fig. 3a.

It is seen from the comparison of the time-dependences of the velocities of the gas and the particles (Fig. 2) that the amplitude of the particle velocity decreases as the particle diameter increases. In this case, the time of particle acceleration and the length of the acceleration section increase. For larger particles (d = 1 mm), in the accelerating section in the presence of gas pulsations the mean Nusselt number becomes smaller than in the case where there are

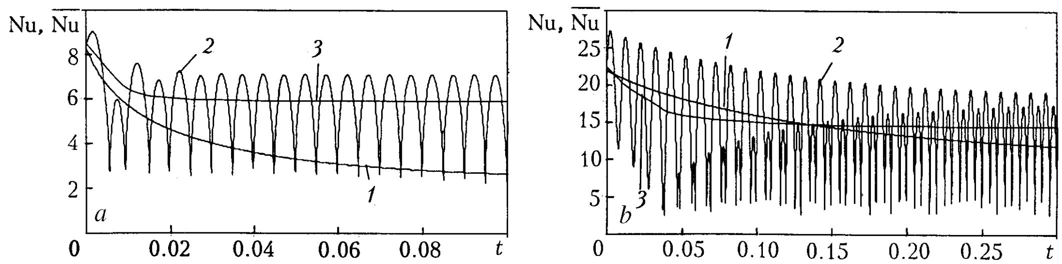

Fig. 3. Dependence of the current (1, 2) and the period-mean Nusselt numbers on time ($\bar{v}_1$ = 60 m/sec): 1) v_1^A = 0; 2, 3) 40 m/sec; a) d = $1{\cdot}10^{-4}$; b) $1{\cdot}10^{-3}$ m.

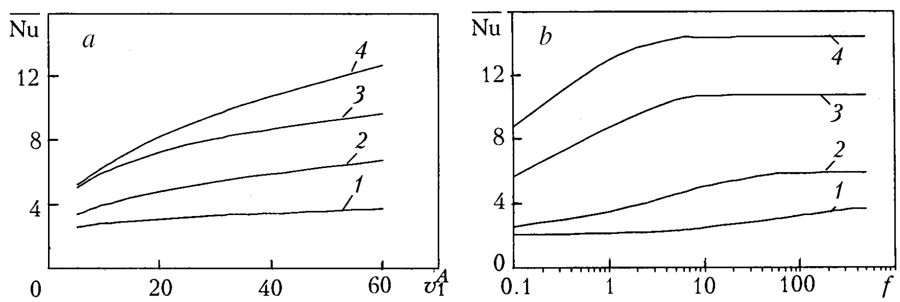

Fig. 4. Dependence of the Nusselt number on the amplitude (a) and frequency (b) of gas-velocity fluctuations ($\bar{v}_1$ = 60 m/sec): a) 1) f = 1 and 2) 100 Hz at d = $1{\cdot}10^{-4}$ m, 3) f = 1 Hz and 4) 100 Hz at d = $5{\cdot}10^{-4}$ m; b) 1) d = $2{\cdot}10^{-5}$, 2) $1{\cdot}10^{-4}$, 3) $5{\cdot}10^{-4}$, and 4) $1{\cdot}10^{-3}$ m at v_1^A = 40 m/sec.

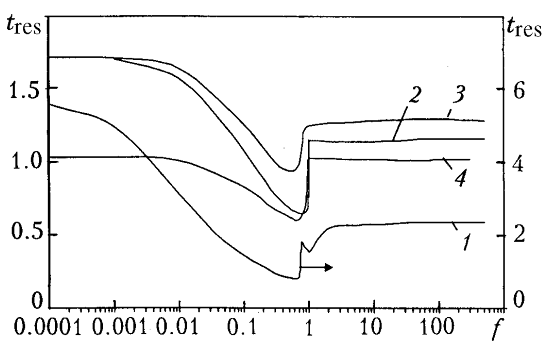

Fig. 5. Dependence of the time of particle residence in the apparatus on the frequency of gas-velocity fluctuations: 1) d = $1{\cdot}10^{-3}$ m, $\bar{v}_1$ = 10 m/sec, and v_1^A = 40 m/sec; 2) $1{\cdot}10^{-3}$, 20, and 40; 3) $1{\cdot}10^{-3}$, 20, and 20; 4) $1{\cdot}10^{-4}$, 20, and 20, respectively.

no gas pulsations. However, in the steady-state oscillatory mode after acceleration of particles the mean Nusselt number increases in the case of gas pulsations and exceeds about 1.5 times the Nusselt number for the case of steady-state gas motion (Fig. 3b).

The dependence of the mean Nusselt number on the amplitude and frequency of the fluctuations of the gas velocity is shown in Fig. 4. It is seen from the analysis that as the amplitude of the gas velocity increases, the mean Nusselt number $\overline{\text{Nu}}$ increases and heat transfer is thus enhanced. With an increase in frequency, the number $\overline{\text{Nu}}$ increases to a certain limiting value and then remains virtually constant. With an increase in the particle diameter the value of the limiting frequency decreases. This is due to the fact smaller particles "follow" the gas flow "more actively" and the relative velocity decreases.

For technological heat exchangers, e.g., pneumopipes, the time of particle residence in the apparatus, i.e., the time of thermoprocessing, is of importance. By virtue of this, we studied the effect of the frequency and amplitude of fluctuations of the gas velocity on the time of flow of the particles until they reach a height of $x_f = 20$ m, i.e., the time of particle residence on the section $[0, x_f]$ (or in a vertical 20-m-long pneumochannel). The dependence of the residence time on the frequency of fluctuations of the gas velocity is presented in Fig. 5, from which it is seen that t_{res} substantially decreases with an increase in the frequency of gas pulsations, then it sharply increases and remains virtually constant. Within the studied range of parameters, the time of particle residence at large frequencies does not exceed its values at small frequencies. For particles with a diameter of 1 mm, a decrease in the amplitude of gas-velocity fluctuations (from 40 to 20 m/sec) leads to a decrease in the residence time (curves 2 and 3). The time t_{res} also decreases with a decrease in the particle diameter and an increase in the mean velocity of the gas flow.

The dependences obtained can be useful in designing apparatuses with pulsed input of gas.

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NOTATION

A, amplitude of gas-phase displacement, m; *c*, heat capacity, J/(kg·K); *d*, particle diameter, m; e_2, internal energy, J/kg; *f*, frequency, Hz; F_n, force of interphase interaction [caused by hydrodynamic drag ($n = \mu$), effect of virtual mass ($n = m$), nonstationarity of the boundary layer around particles (n = B), and the force of gravity ($n = g$)], N/m^3; *g*, free-fall acceleration, m/sec^2; n_2, counted concentration of particles, m^{-3}; *P*, pressure, Pa; q_2 and Q_2, heat fluxes related to one particle and the dispersed mixture, W and W/m^3; S_2, area of the midsection of a particle, m^2; *t*, time, sec; t', period of oscillations, sec; *T*, temperature, K; *v*, velocity, m/sec; *x*, vertical coordinate, m; ε_1, porosity; λ, thermal conductivity; W/(m·K); μ and ν, dynamic and kinematic viscosity of the gas, Pa·sec and m^2/sec; ξ, hydrodynamic drag; ρ, density, kg/m^3; τ, time $(0 \le \tau \le t)$, sec; ω, angular frequency, rad/sec; Nu, Pr, and Re, Nusselt, Prandtl, and Reynolds numbers. Indices: 1 and 2, gas and solid particles, respectively; *A*, amplitude; f, finite value; zero (subscript), initial value, zero (superscript), a value per particle; res, residence; overbar, mean value; *, half-period; m, mass; B, Basset.

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